Department of Mathematics
Pattamundai College, Pattamundai

Core11, 5^{th} Semester

Paper : Multivariate Calculus (Calculus -II)

Section–A

1\. a) If $f(x,y,z) = x^2ye^{2x} + (x+y-z)^2$ then find $\frac{d}{dz}\{f(1,1,z^2)\}$

b) Find the domain of the function $f(x,y,z) = \dfrac{1}{\sqrt{4-x^2-y^2-z^2}}$

c) Evaluate $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2-2xy+y^2}{x-y}$

d) Evaluate $\operatorname{Lim}\limits_{(x,y)\to(0,0)} \dfrac{x+ye^{-x}}{1+x^2}$

e) Write the condition for the function f(x,y) is continuous at the point (x_0,y_0).

f) Find $\lim\limits_{(x,y)\to(5,2)} x\cos \pi y$.

g) Find $f_x(1,x)$ for the function $f(x,y) = \sqrt{x^4+y^4+1}$

h) If $f(x,y,z) = xy^2+yz^3+xyz$ then determine

i) fx (ii) fy (iii) fz

i) What is Harmonic function?

j) Find the equation of tangent plane to the surface $f(x,y) = x^2+y^2+\sin xy$ at the point $P_0(0,2,4)$?

k) Evaluate f(1.01, 2.03) where $f(x,y) = 3x^4 + 2y^4$, using incremental approximation to f.

l) Find the total differential of the function $f(x,y) = 8x^3y^2 - x^4y^5$.

m) If $z = (4+y^2)\,x$, where $x = e^{2t}$ and $y = e^{3t}$ then compute $\dfrac{dz}{dt}$

n) If $z = xy + f(x^2+y^2)$, show that $y\dfrac{\partial z}{\partial x} - x\dfrac{\partial z}{\partial y} = y^2 - x^2$

o) Find $\vec{\nabla} f(x,y)$ for $f(x,y) = \dfrac{y}{x} + \dfrac{x}{y}$

p) Find the slope of the level curve $x^2 + y^2 = 2$ where x=1 and y = 1.

q) If $\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$ then find the unit vector in the direction of $\vec{R}$.

P.T.O.

r) Find the graient of the following functions.

i) $f(x,y) = \ln(x^2+y^2)$ (ii) $g(x,y) = \sin(x+2y)$

s) Find the equation of tangent plane to the surface $x^3+2xy^2-7x^3+3y+1 = 0$ at p_0 (1,1,1)?

t) Define absolute extrema?

u) Discuss the nature of the critical point (0,0) for the surface $f(x,y) = 1-x^2-y^2$?

v) If f(x,yz) be a scalar field then define gradient of f.

w) What is conservative field?

x) What is cross-potential test?

y) When we can say a vector field is incompressible?

z) Find div $\vec{F}$ given that $\vec{F} = \vec{\nabla} f$, where $f(x,y,z) = x^2yz^3$

2. a) Define curl of a vector field $\vec{F}$.

b) Express div ($\vec{F} \times \vec{G}$) in terms of curl $\vec{F}$ and Curl $\vec{G}$.

c) What is Laplace's equation?

d) Evaluate $\iint_R 4dA$, for R ; $0 \le x < 2$, $0 \le y \le 4$.

e) Find the area of the curve $r = 2(1-\cos\theta)$

f) Evaluate $\int_0^{\pi}\int_0^{a\sin\theta} r\,dr\,d\theta$

g) Write the conversion formula spherical to rectangular ie, $(\rho,\theta,\phi,)$ to (x, y, z)?

h) Convert the rectangular coordinate (1,2,3) to spherical.

i) What is the Jacobian of the transformation T defined by x = g(u,v) and y = h (u,v)?

j) Find the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ of $x = e^{u+v}$, $y = e^{u-v}$.

k) Evaluate $\int_c (xdy - ydx)$, for C : $2x-4y = 1$, $4 \le x \le 8$

l) Define $\int_c \vec{F}.\,d\vec{R}$ is independent of path in a region D.

m) Test the vector field $\vec{F} = 2xy\hat{i} + x^2\hat{j}$ is conservative or not?

n) Evaluate $\oint_C \frac{\partial f}{\partial n} ds$, where $f(x,y) = x^2y-2xy+y^2$

o) State Stoke's Theorem?

p) State Green's Theorem?

Section – B

1. a) Show that $\lim\limits_{(x,y)\to(0,0)} \frac{x^3+y^2}{x-y} = 0$ does not exist.

b) Using Polar co-ordinates evaluate $\lim\limits_{(x,y)\to(0,0)} (1+x^2+y^2)^{\frac{1}{x^2+y^2}}$

c) Given that the function $f(x,y) = \begin{cases} \frac{x^3+y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ A, & (x,y) = (0,0) \end{cases}$ is continuous at the origin, What is A?

d) For f(x,y) = cos xy^2, show that $f_{xy} = f_{yx}$.

e) Show that the product function $P(L,K) = L^{\alpha} K^{\beta}$, where α, β are constants, statistics $L\frac{\partial p}{\partial L} + K\frac{\partial p}{\partial k} = (\alpha+\beta)P$.

f) Find the equation for the tangent plane and normal line to the surface $z = \sin x + e^{xy} + 2y$ at the point (0,1,3)?

g) Show that the following function is not differentiable at (0,0) $f(x) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

h) Show that the function f defined by f(x,y) = cos (x+y) is differentiable at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$.

i) Let $z = x^2+3xy+y^2$, where $x = \cos\frac{2\pi}{8}$ and $y = e+ \sin\frac{2\pi}{8}$. Then Compute $\frac{dz}{dt}$, using chain rule.

j) If f is differentiable and $z = u+f(u^2v^2)$, show that $u\frac{\partial z}{\partial u} - v\frac{\partial z}{\partial v} = u$.

k) Let f and g be twice differentiable function of one variable and let u(x,t) = f(x+ct) + g (x–ct) for a constant C. Show that $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$

l) Find the directional derivative of the function $f(x,y) = \sec(xy-y^2)$ at the point $p_0(2,0)$ in the direction of the vector $-\hat{i} - 3\hat{j}$.

m) Let f(x, y, z) = xyz and let $\vec{u}$ be a unit vector perpendicular to both $\vec{V} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{w} = 2\hat{i} + \hat{j} - \hat{k}$. Find the directional derivative of f at $p_0(1, -1, 2)$ in the direction of $\vec{u}$.

n) Find the equations of the tangent plane and normal line to the surface $z = \sin x + e^{xy} + 2y$ at $p_0(0,1,3)$?

o) Find the relative extrema of the function $f(x,y) = y^2+x^2y+x^4$ at (0,0)?

P.T.O.

p) Find all points on the surface $y^2 = 4+xz$ that are closest to the origin?

q) Find $c \in R$, for which the vector field. $F(x,y) = (\sqrt{x} + 3xy)\hat{i} + (cx^2 + 4y)\hat{j}$ is conservative.

r) Find divergence of the vector field $\vec{G}(x,y,z) = (e^{-x} \sin y)\hat{i} + (e^{-x} \cos y)\hat{j} + \hat{k}$ at (1, 3, –2)

s) Let A be a current art vector and let $\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$. show that $\text{div}(\vec{A} \times \vec{R}) = 0$

t) Let A be a constant vector and let $\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$. Show that $\text{curl}(\vec{A} \times \vec{R}) = 2A$

u) Show that curl (grad f) = $\vec{0}$.

v) Show that $f(x,yz) = (x^2+y^2+z^2)^{\frac{1}{2}}$ is harmonic

w) Evaluate the iterated integral $\int_0^2\int_0^1 (x^2+xy+y^2)\,dydx$

x) Computer $\int_0^{2\pi}\int_0^{\pi} \sin(x+y)\,dxdy$

y) Suppose D is the quarter ring with radii r=1 and r =2. Then evaluate $\iint_D (3x+8y^2)\,dydx$

z) Find by double integration the area lying inside the cardiod $r = 1+\cos\theta$ and outside the circle r=1.

2. a) Compute the iterated triple integrals $\int_0^1 \int_{x-1}^{x^2} \int_x^y (x+y)\,dzdydx$

b) Find volume of the ellipsoid $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} = 1$

c) Find the volume of the solid region common to the cylinders $x^2+z^2 = 1$ and $x^2+y^2=1$

d) $\iiint_D (x^2y + y^2z)\,dv$, Where D : $1 \le x \le 3$, $-1 \le y \le 1$, $2 \le z \le 4$.

e) Determine the new region that we get by applying the given transformation to the region R, R is the ellipse $x^2 + \frac{y^2}{36} = 1$ and the transformation is $x = \frac{u}{2}$, $y = 3v$.

f) Find the Jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ of the change of variables $u = 2x - 3y + z$, $v = 2y - z$, $w = 2z$.

g) Evaluate $\int_C (x^2+y^2)\,ds$, for C : $x = e^{-t}\cos t$, $y = e^{-t}\sin t$, $0 \le t \le \frac{\pi}{2}$

h) Evaluate $\int_C \vec{F}.\,d\vec{R}$ where C is any smooth path connecting A(0,0) to B (1,1) If $\vec{F} = (2x-y)\hat{i} + (y^2-x)\hat{j}$.

i) Show that the vector field $\vec{F} = (20x^3z + 2y^2)\hat{i} + 4xy\hat{j} + (5x^4+3z^2)\hat{k}$. is conservative and find a scalar potential function of $\vec{F}$.

j) Find the area of the region enclosed by the curve C : $x = a\cos^3 t$, $y = a\sin^3 t$, for $0 \le t \le 2\pi$.

k) Evaluate $\oint_C \{(x^2-3y)\,dx + 3xdy\}$, where, C is the circle $x^2+y^2 = 4$.

l) Define (i) Simple Curve (ii) Positive Orientation.

m) Find the surface area of the hemisphere $x^2+y^2+z^2=1$ lying above the elliptical region $x^2 + y^2 \leq 1$.

n) Using Stoke's theorem, evaluate $\iint_S (\text{curl}\,\vec{F}.\vec{N})\, ds$, for the vector field $F = 2\hat{z}i + 3\hat{x}j + 5\hat{y}k$ and S is the part of the paraboloid $z = 4-x^2-y^2$ with $z \geq 0$ Use upward unit normal vector.

Section – C

1. a) Show that the function f(x,y) is continuous at the origin, where $f(x,y) = \begin{cases} \dfrac{x^3-y^3}{x^2+y^2} & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$,

b) Verify that $T(x,t) = e^{-t}\cos\frac{x}{c}$ satisfies the heatequation $\frac{\partial T}{\partial t} = c^2 \frac{\partial^2 T}{\partial x^2}$

c) Suppose f (x,y) is differentiable at(3,4) with $f_x(3,4) = 2$ and fy (3,4) =–1. If f(3,4) =5, estimate the value of f(3.01, 3.97)

d) If z = f(x,y) has continuous second order partial derivatives and $x = e^r\cos\theta$, $y = e^r\sin\theta$, show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} = e^{-2r}\left\{\frac{\partial^2 z}{\partial r^2} + \frac{\partial^2 z}{\partial \varphi^2}\right\}$$

e) Find the equation for the tangent plane to the hyperboloid of one sheet $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ at the point $P_0(x_0, y_0, z_0)$?

f) Find all critical points of f(x,y) and classify as a relative maximum, a relative minimum or a saddle point.

$f(x,y) = (x-1)(y-1)(x+y-1)$

g) Find all points in the plane $x+2y+3z = 4$ in the Ist octant where $f(x, y, z) = x^2yz^3$ has a maximum value.

h) Find the distance from the origin to the plane $x+2y+2z = 3$ using a geometric argument.

i) Show that curl (curl $\vec{F}$) = grad div $\vec{F}$ – laplacian $\vec{F}$

j) Show that $\int_0^1\int_0^1 \left(\frac{y-x}{(x+y)^3}\right) dydx$ and $\int_0^1\int_0^1 \left(\frac{y-x}{(x+y)^3}\right) dxdy$ have different values.

k) Show by double integration that the area between the curves $y^2=x$ and $x=y$ is $\frac{1}{3}$.

l) Find the area included between $r = 3\cos\theta$ and $r = 2 - \cos\theta$.

m) Integrate the function $f(x,y,z) = (x^2+y^2)z^2$ over the cylindrical regian D given by $x^2+y^2 \leq 1$, $-1 \leq z \leq 1$.

n) Find the volume of the solid D bounded above by the sphere $x^2+y^2+z^2 = 4$ and below by the cone $z = \sqrt{x^2+y^2}$, using spherical co-ordinates.

P.T.O.

o) Evaluate $\int \vec{F} . \vec{T}\ ds$ where $\vec{F} = < -3y, 3x, 3x >$ and C is the straight line segment from (0, 0, 1) to (1, 1, 1).

p) Find the scalar potential function f for the conservative vector field.

$$F = \frac{kx}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}\hat{i} + \frac{ky}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}\hat{j} + \frac{kz}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}\hat{k} \text{ where k is a constant}$$

q) Compute the flux of the vector field, $F(x, y, z) = 3z^2\,\hat{i} + 6\,\hat{j} + 6xz\,\hat{k}$ across parabolic cylinder S given by $y=x^2$, $0 \le x \le 2$, $0 \le z \le 3$.

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